Quantum Simulation with Rydberg Atoms

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Outline

Dissipative quantum state engineering

Rydberg atoms

Mesoscopic Rydberg gates

A Rydberg Quantum Simulator
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A Rydberg Quantum Simulator
Tailored quantum states are an important resource (quantum simulation, quantum communication, quantum metrology, ...)

Previously: coherent evolution (adiabatic following, quantum logic gates)

New tool: controlled dissipation

HW et al., Nature Phys. 6, 382 (2010)

Engineer a suitable attractor state of the dynamics

Inherently more robust
Open quantum system described by a quantum master equation (Lindblad form)

\[
\frac{d\rho}{dt} = -i\hbar [H, \rho] + \sum_n \gamma_n \left( c_n \rho c_n^\dagger - \frac{1}{2} \{c_n^\dagger c_n, \rho\} \right)
\]

\[\rho = \sum_i p_i |\psi_i\rangle \langle \psi_i|\]

- \(\rho\): Density operator
- \(\frac{d\rho}{dt}\): Time derivative of density operator
- \(H\): Hamiltonian
- \(c_n\): Quantum jump operators (non-Hermitian)
- \(\gamma_n\): Decay rate
Stationary state

- Sufficient (and usually also necessary) condition: \( \frac{d\rho}{dt} = 0 \)
- Special case: pure state \( \rho = |\psi\rangle \langle \psi| \)
- More general: von Neumann entropy

\[
S = - \text{Tr} \{ \rho \log \rho \}
\]

- Only Hermitian jump operators
  \( \Rightarrow \) Maximally mixed state \( (S = \log d) \)

\[
\rho = \begin{pmatrix}
\frac{1}{d} & \frac{1}{d} \\
\frac{1}{d} & \ddots \\
\frac{1}{d} & 0 & \frac{1}{d} & \ddots
\end{pmatrix}
\]

\[
\frac{d\rho}{dt} = -i\hbar [H, \rho] + \sum_n \gamma_n \left( c_n \rho c_n^\dagger - \frac{1}{2} \{c_n^\dagger c_n, \rho\} \right)
\]
Coherent quantum simulation

- Goal: simulate an effective plaquette interaction
  \[ H = A_p = \sigma_x^{(1)} \sigma_x^{(2)} \sigma_x^{(3)} \sigma_x^{(4)} \] using the \( N \)-body gate
- Map \( |\pm 1\rangle \) eigenstates of \( A_p \) onto \( |0\rangle, |1\rangle \) of the control atom

\[
R_y = \exp(-i\pi\sigma_y/4) = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & -1 \\
1 & 1
\end{pmatrix}
\]
Coherent dynamics

- Apply the mapping $G$ transferring the eigenvalue of $A_p$ onto the control spin
- Write a phase $\exp(-i\phi \sigma_z)$ onto the control spin
- Undo the mapping $G = G^{-1}$

\[
e^{-iH_\tau/\hbar} = G G^\dagger e^{-i\phi \sigma_z}
\]

- Simulates the Hamiltonian $H$ at discrete times $t = k\tau$ (digital)
- Energy scale $E_0 = \hbar \phi / \tau$

HW et al., Nature Phys. 6, 382 (2010)
Dissipative cooling

- Goal: cool into the ground state of $H = A_p (-1 \text{ eigenstate})$
- Use the same mapping (ensemble $\mapsto$ control) as before
- Instead of writing a phase on the control spin: controlled spin flip of one random ensemble spin $j$

$$U = |0\rangle\langle 0|_c \otimes 1 + |1\rangle\langle 1|_c \otimes \exp(i\phi\sigma_z^{(j)})$$

- If we do a spin flip: control atom will not end in $|0\rangle$
- Reset spin (incoherent) from $|1\rangle$ to $|0\rangle$
- Discrete Markovian master equation

$$\dot{\rho}(t+\tau) = \dot{\rho}(t) + \gamma \left( c\dot{c}^\dagger - \frac{1}{2} \left\{ c^\dagger c, \dot{\rho} \right\} \right)$$

- Rate $\gamma = \phi^2/\tau$, jump operator $c = \sigma_z^{(j)}(1 + A_p)/2$
Scaling up

- Cooling: random walk of the anyons
- Averaging over $10^3$ realizations of the dynamics
- Imperfections: residual anyon density $n$

![Graph showing the decay of $n$ over time $t[\gamma^{-1}]$]
Linear response theory

- Gate error probability \( \varepsilon \): probability to end up in a state orthogonal to the desired one
- Toric code: gate errors create anyons

Uncorrelated errors \( \Rightarrow \) Effective temperature

\[
T \approx -\frac{2E_0}{k_B \log n}
\]

- Anyon density \( n \) within linear response: \( n = 14\varepsilon \)

\( \Rightarrow \) Effective temperature benchmarks the quantum simulator

Experimental realization

- Proof of principle experiment with trapped ions
- $N$-body Mølmer-Sørensen gate
- Four ensemble spins + 1 control ion
- Minimal instance of a toric code Hamiltonian (1 plaquette)

J. Barreiro et al., Nature 470, 486 (2011)
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Rydberg states

- Electronic excitations with large principal quantum number $n$
- Hydrogen-like wavefunctions

Rb ground state $5s$: Rydberg state $35s$: 
Properties of Rydberg atoms

- Long ($\propto n^3$) life-time (100 $\mu$s)
- Large ($\propto n^2$) diameter (100 nm)
- Highly sensitive to electric fields
- Strong dipolar ($\propto n^4$) or van der Waals ($\propto n^{11}$) interactions

Interaction strength:

- BEC
- Collisions
- Magnetic Atoms
- Polar Molecules
- Rydberg Atoms
Quantum defect

- Correction to the hydrogenic energy levels

\[ E = -\frac{Rhc}{(n - \delta_{l,j})^2} \]

- Depends only on the angular quantum numbers
- Lifts \( l \) degeneracy

Van der Waals

\[ |np\rangle \]

|ns\rangle

Ising interaction \( \sigma_z \sigma_z \)

Resonant dipole-dipole

|np\rangle

|n - 1p\rangle

Flip-flop terms \( \sigma_+ \sigma_- \)
Rydberg blockade

- Strong interaction between Rydberg states shift the doubly excited state
- Rabi oscillations are enhanced by a factor $\sqrt{N}$

D. Jaksch et al., PRL 85, 2208 (2000)
M. D. Lukin et al., PRL 87, 037901 (2001)

see also M. Saffman

$\Omega_{\text{eff}} = (1.38 \pm 0.03)\Omega$
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Single qubit gates

- Encode quantum information into two hyperfine ground states
- Resonant microwave driving

\[ H = \hbar \omega |1\rangle\langle 1| + \hbar \Omega \cos(\omega t)(|0\rangle\langle 1| + |1\rangle\langle 0|) \]

- Rotating frame

\[ |1\rangle \rightarrow |1\rangle \exp(i\omega t) \]

\[ H = \frac{\hbar \Omega}{2}(1 + \exp(2i\omega t))(|0\rangle\langle 1| + |1\rangle\langle 0|) \]

- Rotating wave approximation: neglect fast oscillating term
- Unitary time-evolution operator

\[ U = \exp(-iHt/\hbar) = \begin{pmatrix} \cos(\Omega t) & -i \sin(\Omega t) \\ -i \sin(\Omega t) & \cos(\Omega t) \end{pmatrix} \]

- Rotation about $x$ axis; rotation about $z$ axis using detuning
Many-body quantum gates

- Introduce auxiliary atoms to mediate interactions
- Controlled-NOT\(^N\) plus single qubit gates
- Single-step implementation based on Electromagnetically Induced Transparency
  
  M. Müller, I. Lesanovsky, HW, H.P. Büchler, P. Zoller, PRL 102, 170502 (2009)
Electromagnetically Induced Transparency

- Destructive interference effect in a multi-level system

\[ E \quad |2\rangle \]

\[ \Omega_c \]

\[ \Omega_p \]

\[ |1\rangle \]

\[ |0\rangle \]

\[ |1\rangle + |2\rangle \]

\[ |1\rangle - |2\rangle \]

- Linear susceptibility \( \chi \sim \rho_{01} \)

Absorption

Optical Frequency

Re \( \chi \)

\( \delta \)
Pulse sequence

- Ensemble atoms start in \(|A\rangle\)
- 3 laser fields: \(\Omega_r\), \(\Omega_c\) (strong), \(\Omega_p\) (weak)

1. \(\pi\) pulse on control atom \(|1\rangle \mapsto |r\rangle\)
2. Adiabatic Raman pulse from \(|A\rangle\) to \(|B\rangle\)
   - Control in \(|0\rangle\): transfer is blocked (EIT)
   - Control in \(|r\rangle\): transfer is enabled
3. \(\pi\) pulse on control atom
Linear susceptibility

- Effective Hamiltonian:

\[ H \sim \sqrt{2} \left( \frac{\Omega_p}{\Omega_c} \right)^2 |+\rangle \langle +| + (1 + V) |R\rangle \langle R| + \frac{\Omega_p}{\Omega_c} \left( |+\rangle \langle R| + \text{h.c.} \right) \]

- Rydberg-Rydberg interaction shifts the EIT resonance
- Two-photon resonance (\( \delta = 0 \)) is no longer dark
Limiting factors

- Incoherent processes
  - Radiative decay of the $|P\rangle$ level: $\Delta \gg \gamma_P$
  - Radiative decay of the control atom: $\Omega_p \gg \gamma_r$

- EIT condition
  - Sharp EIT resonance: $\Delta \gg \Omega_c$
  - Interaction shift: $V \gg \Omega_c$

- Adiabatic Raman pulse
  - $\Omega_c \gg \Omega_p$

- Effect of ensemble-ensemble interaction $V_{ee}$ depends on $N$

- Transfer is enhanced by a repulsive interaction

- Blocking is reduced: superatom calculation gives a phase shift

$$\phi < N(N - 1)\phi_0 \left(1 - \frac{2}{V_{ee}}\right); \quad \phi_0 = \frac{35\pi}{48}(\Omega_p/\Omega_c)^2$$
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Current status

- Mott insulator of ultracold atoms in optical lattices with more than 1,000 atoms
- Arbitrary lattice geometries
- Single site addressing
- Need to achieve strong interactions between the atoms
- Laser excitation to Rydberg states

(M. Greiner, Harvard) (I. Bloch, MPQ)
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(M. Greiner, Harvard)
Nature 491, 87 (2012)
(I. Bloch, MPQ)
Example I: Toric code

- Control atoms (red) and ensemble atoms (blue) on a 2D lattice
- Plaquette interaction $A_p = \sigma^n_x$ (light red), site interaction $B_s = \sigma^n_z$ (green)

- Toric code Hamiltonian (Kitaev, 2003)

$$H = -E_0 \left( \sum_p A_p + \sum_s B_s \right)$$

- Rydberg quantum simulator: $E_0 \approx 100$ kHz
- Cooling: random walk of the anyonic excitations
Example II: Lattice Gauge Theory

- \[ H = U \sum_o \left( \sum_{i \in o} \hat{\sigma}^{(i)}_z \right)^2 - J \sum_p B_p + V \sum_p B_p^2 \]
- Ring-exchange \( B_p = \hat{\sigma}_+ \hat{\sigma}_- \hat{\sigma}_+ \hat{\sigma}_- + h.c \) via gate sequence
- Low-energy sector \( (U \gg J, V) \): three spins up/down on each octahedron
- \( V = J \): Rokhsar-Kivelson point (non-stabilizer state)
- \( V < J \): Spin liquid phase with Coulombic \( 1/r \) interactions
Example III: Fermi-Hubbard model

- 2D Fermi-Hubbard model is believed to be realized in high-temperature cuprate superconductors

\[ H = -t \sum_{<ij>\sigma} c_i^\dagger c_j + U \sum_i n_i^\uparrow n_i^\downarrow \]

- Mapping fermions onto spins: Jordan-Wigner transformation

- Problem in 2D: Wigner strings (highly nonlocal interactions)

- Solution: Introduce auxiliary fermion field


\[ H_{\text{aux}} = -V \sum_{\{i,j\}\sigma} P_{i',j'} P_{j'+1,i'-1} \]

\[ P_{i',j'} = (d_{i'\sigma} + d_{i'\sigma}^\dagger)(d_{j'\sigma} - d_{j'\sigma}^\dagger) \]

- Results in local six-body interactions
Summary and outlook

- Mesoscopic Rydberg gate based on Electromagnetically Induced Transparency
- Universal quantum simulation on a 100 kHz timescale
- Dissipative state preparation
- Simulation of complex spin models
- Robust against real experimental errors

HW et al., Nature Phys. 6, 382 (2010)
Herrenhausen Castle
PhD and Postdoc Positions Available!

- Freigeist project “Quantum States on Demand”
- Quantum state engineering
- Dissipative many-body quantum dynamics

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