Quantum Galvanometer by Interfacing a Vibrating Nanowire and Cold Atoms

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ABSTRACT: We evaluate the coupling of a Bose–Einstein condensate (BEC) of ultracold, paramagnetic atoms to the magnetic field of the current in a mechanically vibrating carbon nanotube within the frame of a full quantum theory. We find that the interaction is strong enough to sense quantum features of the nanowire current noise spectrum by means of hyperfine-state-selective atom counting. Such a nondestructive measurement of the electric current via its magnetic field corresponds to the classical galvanometer scheme, extended to the quantum regime of charge transport. The calculated high sensitivity of the interaction in the nanowire–BEC hybrid systems opens up the possibility of quantum control, which may be further extended to include other relevant degrees of freedom.

KEYWORDS: Carbon nanotubes, nanowires, cold atoms, Bose-Einstein condensates, quantum sensors, quantum noise

Carbon nanotube (CNT) technology has evolved to the point where CNTs can be produced with a variety of mechanical and electrical properties.1 Besides applications in chemical,2 biological,3 and mass sensing with up to single atom resolution,4 high-quality nanomechanical resonators,5 one-dimensional electric transport effects, and their coupling to mechanical motion of the CNTs have been observed.6,7 The increasingly fine control of these degrees of freedom anticipates the manipulation of CNTs at a quantum mechanical level that has been recently achieved in other nanomechanical systems.8

Atomic physics has undergone a breathtaking evolution since laser cooling and trapping techniques allowed for the preparation of localized and isolated atom samples.9 The thermal noise has been reduced to the ultimate quantum noise level where the ultracold atoms (below 1 μK) form a degenerate quantum gas, called a Bose–Einstein condensate (BEC).10 This cloud of atoms can be controlled in all relevant degrees of freedom with an unprecedented precision. The cloud can be positioned on the submicrometer scale in magnetic11 or optical traps.12 The internal electron dynamics can be driven by external laser or microwave fields, which allow for the precise preparation as well as the high-efficiency detection of the electronic state. The system of trapped neutral atoms in the collective BEC state is an ideal probe of external fields.13

The first attempts to make a BEC interact with a CNT have been reported only recently. Atom scattering from the CNT’s van der Waals potential14 and the field ionization due to charged suspended nanotubes have been observed.15 Recent proposals have discussed how to make use of the CNT as a current-carrying thin nanowire to tighten the magnetic trapping potential for cold atoms16 and how to form nanoscale plasmonic atom traps along silver-decorated CNTs.17 The integration of CNTs and atomic BECs opens up new avenues toward hybrid systems coupling these objects at a quantum level in a controlled way. One can envisage the coherent interfacing of very different degrees of freedom such as electronic, mechanical, and spin variables. A variety of novel nanodevices for precision sensing, quantum measurement, and quantum information processing could be developed on the basis of CNT-BEC coupling. The question is whether there is a suitable interaction between selected degrees of freedom and whether the cross-coupling is strong enough to design useful quantum devices.

In this Letter we theoretically evaluate the interaction between the current through the nanowire and atoms in a condensate. We describe how the internal atomic dynamics in the hyperfine states couples to the magnetic field generated by the CNT current. Starting from a fully quantum model, we calculate the time evolution of the atomic system and construct a scheme to measure the current via an atomic observable. All this leads to the conclusion that quantum dynamical properties of the CNT are detectable by making use of the mature technology of ultracold atoms. A more general implication is that the other degrees of freedom of the CNT and BEC that take part in the dynamics can also be accessed and possibly manipulated in variants of this scheme.

For the sake of concreteness, we focus on the measurement of the current through the CNT in the galvanometer scheme,
i.e., when the electric current is sensed in a nondestructive way via its magnetic effect. We are interested in the quantum transport limit of a mesoscopic conductor. In principle, a quantum object such as a spin-1/2 particle precessing in the magnetic field is sensitive to the quantum properties of the current. The full counting statistics of the charge transport process\textsuperscript{18} can be reconstructed from the density matrix of the spin.\textsuperscript{19} This scheme is sometimes referred to as the "quantum galvanometer."\textsuperscript{20} However, its realization has to meet several conditions (stability, sensitivity, and detectability; see below), which has seemed out of the question so far.

In our scheme, the fictitious spin of the quantum galvanometer is realized by a cloud of long-term trapped atoms. One can make use of the collective BEC state, which greatly enhances the sensitivity of the internal hyperfine dynamics to external magnetic fields.\textsuperscript{21} Moreover, a direct readout method for the spin state is available since the populations in the magnetic sublevels can be counted by state selective ionization with single-atom resolution.\textsuperscript{22} The populations are expressed in the diagonal elements of the spin density matrix. In principle, the off-diagonal elements could also be measured, which is required to determine the generating function of the full counting statistics. Here we propose a simpler measurement restricted to the diagonal elements, since it is already enough to characterize some of the quantum features of the current. In particular, we will show that the ordinarily defined current noise spectrum

\[
S(\omega) = \int_{-\infty}^{\infty} d\tau e^{i\omega \tau} \langle \hat{I}(0) \hat{I}(\tau) \rangle
\]

is directly related to the time evolution of the populations in the given magnetic sublevels. The proposed galvanometer allows for the measurement of the spectrum by scanning the adjustable variable \(\omega\), which spans both the negative and positive frequency ranges. Asymmetry of the measured spectrum around \(\omega = 0\) would reveal quantum features.\textsuperscript{23} This is due to the fact that asymmetry of \(S(\omega)\) is equivalent with the noncommutativity of the current operator at different instances, as one can easily check in the above equation.

The scheme is presented in Figure 1 for a possible architecture of building the galvanometer on an integrated platform, the so-called "atomchip."\textsuperscript{11,24} It is a compact, substrate-based electric circuit of currents that create highly tunable magnetic traps for the neutral atoms. The chip may support a contacted CNT on the side facing the BEC. The suspended CNT is a high-Q mechanical oscillator,\textsuperscript{5,8,25,26} which is driven to oscillate coherently with large amplitude. We note that, in order to simplify our calculations, here we assume that the vibration of the CNT is driven by a mechanical source (e.g., a piezo crystal) instead of electric fields. Therefore the CNT consists in creating the resonance conditions such that the low-frequency components of the current noise spectrum be close to resonance with the hyperfine transitions (Zeeman splitting), c.f., Figure 2. Initializing the atoms in the BEC in a well-defined hyperfine state, the number of atoms transferred to another state follows a statistics determined by the low-frequency part of the current noise spectrum. The noise source can be arbitrary, e.g., thermal or intrinsic quantum noise; for generality, we will represent the current as a quantum operator in the derivation. Finally, the measurement is accomplished by detecting the number of transferred atoms in a given time period, which can be performed by means of state-selective ionization technique at the single-atom resolution level.\textsuperscript{22}

To be specific, we consider ultracold \(^{87}\)Rb atoms in the hyperfine state \(F = 1\). The atomic spin \(\vec{F}\) interacts with the magnetic field according to the Zeeman term \(H_z = g_\mu_B \vec{B} \cdot \vec{F}\), where \(\mu_B = e\hbar/2m_F\) is the Bohr magneton, the Landé factor is \(g_F = -1/2\), and \(\vec{F}\) is measured in units of \(\hbar\). The dominant component of the magnetic field is a homogeneous offset field \(B_{\text{ext}}\) in the \(z\) direction. The eigenstates of the spin component \(F_z\) are then well separated by the Zeeman shift. These are the magnetic sublevels labeled by \(m = -1,0,1\). On top of the offset field, there is an inhomogeneous term that creates an ellipsoidal potential around the minimum of the total magnetic field. The atoms are then subject to the static potential \(V_m, r) = -mV_z(r) + g_\mu_B m B_{\text{ext}} r^2\), where \(V_z(r) = M/2[\alpha_x^2(x^2 + y^2) + \alpha_y^2z^2]\), with \(M\) being the atomic mass. The potential is diagonal on the \(F_z\) basis, and is confining only for \(m = -1\).
Let us consider a single CNT of length $L$ that is electronically contacted and carries a current $I(t)$. It generates a magnetic field that interacts with the atomic spin. Similar coupling has been considered\textsuperscript{27} between a vibrating nanomagnet and a BEC. The CNT is aligned with the $z$ axis (see Figure 1) having a mean distance $y_0$ from the condensate. This distance is large enough ($y_0 \geq 1 \mu m$ to avoid van der Waals-type interactions between the atoms and the CNT\textsuperscript{14,15,28–30}). The CNT is driven to mechanically oscillate harmonically at an angular frequency $\omega_m$ in the $10 MHz$ range\textsuperscript{25,26} and with an amplitude $a$ in the $y$-$z$ plane ($a \ll y_0$). The resulting time-dependent magnetic field $\vec{B}_{\text{CNT}}(t)$ is an operator, since its source is the current operator $I$. With a proper tuning of the Zeeman splitting via the offset $B_{\text{off}}$, the $x$ and $y$ components of $\vec{B}_{\text{CNT}}$ can quasi-resonantly generate transitions between the magnetic sublevels $m$. Such a transfer of trapped atoms into untrapped ones is the underlying mechanism of the radio frequency ($rf$) outcoupler of an atom laser.\textsuperscript{31} Here a fast detection of the $m$-component $\Psi_m(r,t)$ of the condensate wave function can be approximated as $\Psi_m(r,t) \approx \psi_m(r,t)$, where $\Psi_m(r,t)$ is obtained as

$$\psi_m(r,t) = \mathcal{D}(r) \int \frac{d^3r}{(2\pi)^3} \frac{1}{\sqrt{\mathcal{N}}} \psi_m(r) \left[ \left( -\frac{\hbar^2 \nabla^2}{2M} + V_m(r) \right) \mathcal{D}(r) \right] \psi_m(r)$$

(2)

where the first term is the kinetic energy. The atom–atom interaction in this ultracold regime is $s$-wave scattering, and spin flipping collisions are negligible. In principle, the coupling term induces a dynamical back action on the current and the vibration of the CNT. Here we will disregard the back action; however, we note that it could be the source of interesting new effects in similar schemes.

We assume that there is a condensate of a number of $N$ atoms in the trapped $m = -1$ magnetic sublevel. For convenience, we separate the condensate part $(N_r/2)\phi_{\text{BEC}}$ from the excitations $(\delta \Psi_m(r))$, which will be treated perturbatively. The condensate wave function is $\phi_{\text{BEC}}(r,t) = e^{-\phi_{\text{BEC}}(r)}$ where $\mu$ is the chemical potential, and $\phi_{\text{BEC}}(r)$ obeys the Gross–Pitaevskii equation.\textsuperscript{31} We restrict ourselves to the Thomas–Fermi approximation, i.e., we neglect the kinetic energy term in the Gross–Pitaevskii equation. Then the condensate wave function is

$$\phi_{\text{BEC}}(r) = \sqrt{\frac{\mu - V_t(r)}{N_g}},$$

(3a)

where

$$\mu = \mu - \frac{1}{2} \hbar B_{\text{off}} = \left( N_g \frac{15}{8\pi} \alpha \omega_m \right)^{2/5} \left( \frac{M}{2} \right)^{3/5}$$

(3b)

The condensate shape is an ellipsoid with principal semiaxes $b = (2\mu/Ma^2)^{1/2}$ and $c = (2\mu/Ma^2)^{1/2}$. The atom–atom collisions are accounted for by $g = 4\pi \hbar^2 a/M$, where $a$ is the $s$-wave scattering length ($a = 5.4$ nm for $^{87}$Rb). We will neglect the back action of the other spin components onto the condensate and assume that the condensate is intact during the interaction time.

To first order in the perturbations, the equation of motion for the component $\delta \Psi_m$ after some straightforward calculation, is obtained as

$$i\hbar \frac{\partial}{\partial t} \delta \Psi_0(r,t) = \left( \frac{-\hbar^2 \nabla^2}{2M} + N_g \phi_{\text{BEC}}(r,t)^2 \right) \delta \Psi_0(r,t)$$

$$- \frac{\mu_B}{2\sqrt{2}} (\vec{B}_x - i\vec{B}_y) (\sqrt{N} \phi_{\text{BEC}}(r,t) + \delta \Psi_{-1}(r,t))$$

(4)

where we have omitted the subscript "cnt" when writing the components of the magnetic field created by the current through the nanotube, which for $a \ll y_0$ can be approximated as $\vec{B}_x(r,t) \approx B_0(x,t) \cos(\omega_{\text{CNT}} t)$, $i = (x,y)$. The last term with the quantum field $\delta \Psi_{-1}$ in eq 4 is small compared to that of the condensate part, and will be neglected. In accordance with the Thomas–Fermi approximation of the condensate, we neglect the kinetic energy of the excited field, too. By moving to a frame rotating at the frequency $\mu'/\hbar = \omega_{\text{CNT}}$ we get a simple, spatially local driving equation:

$$\frac{\partial}{\partial t} \delta \Psi_0(r,t) = - \frac{i}{\hbar} \Delta(r) \delta \Psi_0(r,t) + \eta(r) B_{\text{off}}(r,t)$$

(5a)
with a spatially inhomogeneous detuning
\[ \Delta(r) = \frac{1}{\hbar} \left( \hbar \omega_{\text{cnt}} - \frac{1}{2} \mu_B B_{\text{offs}} - V_T(r) \right) \] (5b)
and driving amplitude
\[ \eta(r) = i \sqrt{N} \phi_{\text{BEC}}(r) \frac{\mu_0 \mu_B}{16\pi \sqrt{2} \hbar^2} \frac{a}{y_0} U(r) \] (5c)
The time independence of the driving is due to neglecting all terms that oscillate with \( \omega_{\text{cnt}} \) or \( 2\omega_{\text{cnt}} \) in the rotating frame and average out on time scales longer than \( 1/\omega_{\text{cnt}} \). The magnetic fields \( B_{\text{offs}} \) also average out. The dimensionless function \( U(r) \) expresses the spatial variation of the magnetic field modulation due to the CNT:
\[ U(r) = \int_0^T \frac{x^2 - 2(1 + y)^2 + \left( \frac{1}{2} z^2 - z \right)^2 - \mu(x+y)}{x^2 + (1+y)^2 + \left( \frac{1}{2} z^2 - z \right)^2} \, dt \] (6)
where all the length quantities in the integrand are in units of the CNT−BEC distance \( y_0 \). We note that \( U(r) \) is obtained from an infinitely thin finite-length current-carrying wire that is oscillating as a string clamped at both ends.

Starting with a pure condensate at \( t = 0 \), and letting the system evolve to \( t = T \) (the measurement time), the integral of eq 5a leads to
\[ \delta \Psi_0^*(r, T) = \int_0^T \eta(t) i \hat{I}(T - t) e^{-i \Delta(t)} \, dt \] (7)
which expresses the relation between the quantized current \( \hat{I} \) in the CNT and the atom field in the magnetic sublevel \( m = 0 \). Atom counting in this sublevel allows us to extract quantum statistical properties of the current. We assume stationary current, i.e., \( \langle \hat{I}(r') \hat{I}(r) \rangle = \langle \hat{I}(0) \rangle \langle \hat{I}(r' - r) \rangle \). Then the spatially integrated mean number of atoms transferred into the sublevel \( m = 0 \) during the measurement time \( T \) is
\[ N(\Omega) = \int d^3 r \langle \delta \Psi_0^*(r, T) \delta \Psi_0^*(r, T) \rangle = T \int_{-\infty}^{\infty} d\tau e^{i \Omega \tau} \langle \hat{I}(0) \rangle \langle \hat{I}(\tau) \rangle \langle \hat{I}(\tau) \rangle \] (8)
The transferred atom number is, being explicitly indicated, is a function of the frequency \( \Omega = \omega_{\text{cnt}} - \mu_B B_{\text{offs}}/(2\hbar) \), which can be finely tuned by the magnetic field \( B_{\text{offs}} \). Note that \( \omega_{\text{cnt}} \) is typically around \( 2\pi \times 50 \) MHz, whereas the Larmor frequency \( \mu_B B_{\text{offs}}/(2\hbar) \) can be tuned in the range of 0.1−100 MHz.

The measurable \( N(\Omega) \) is related to the current noise spectrum \( \delta (\omega) \) by a convolution with the spectral resolution function, \( \mathcal{F} \{ (\hat{I}(\tau) \delta \Omega) \} \), where \( \mathcal{F} \{ \ldots \} \) denotes Fourier transform. The mapping involves a triangular pulse function
\[ f(\tau) = \begin{cases} 
1 & \text{if } |\tau| \leq T \\
0 & \text{otherwise}
\end{cases} \] (9)
which originates from the finite measurement time. All properties of the BEC−CNT coupling are embedded in
\[ \mathcal{D}(\tau) = \int d^3 r \eta(r)^2 e^{-i \tau V_T(r)/\hbar} \] (10)
Because of the exponential term, the variation range of the potential energy \( V_T(r) \) determines the intrinsic bandwidth of the BEC as a probe system (see also Figure 3). This bandwidth is the chemical potential \( \mu \), which is typically in the range of kHz for a BEC on a chip.

For very short measurement time \( T \ll \hbar/\mu \), the exponential in the integrand of eq 10 can be approximated by 1, so the spectral resolution is dominated solely by \( f(\tau) \). On the other hand, for long measurement time \( T \gg \hbar/\mu \sim 1 \) ms, the approximation \( f(\tau) = 1 - l(|\tau|/T) \approx 1 \) holds in eq 8 since the function \( \mathcal{D}(\tau) \) introduces a cutoff at about \( \hbar/\mu \). In this limit, the CNT-BEC coupling function \( \mathcal{D}(\tau) \) determines the quantum efficiency of the scheme.

\( \mathcal{D}(\tau) \) can be approximated by Neglecting the variation of the magnetic field within the condensate, \( U(r) \approx U(0) \equiv U \). This leads to the Fourier transform
\[ \tilde{\mathcal{D}}(\omega) \equiv \mathcal{F} \{ \mathcal{D}(\tau) \} = n_{\text{det}} \tilde{d}(\omega) / \mu \] (11a)
\[ n_{\text{det}} = \left[ \frac{\mu_0 \mu_B}{16\pi \sqrt{2} \hbar^2} \frac{a}{y_0} \right] U^2 N \] (11b)
\[ \tilde{d}(\omega) = \begin{cases} 
15/4 & \text{if } \omega \leq 1 \\
0 & \text{otherwise}
\end{cases} \] (11c)
where the normalization \( \int_{-\infty}^{\infty} d\omega \tilde{d}(\omega) = 1 \) is obeyed. In Figure 3, the approximate \( \mathcal{D}(\omega) \) (dotted curve) is compared to exact ones which are obtained numerically for different atom numbers \( N \) when the CNT length and the CNT−BEC distance is fixed. It can be seen that eq 11a gives the correct order of magnitude and shape of the exact \( \mathcal{D}(\omega) \) for a broad range of the BEC size.

The detectable atom number spectrum, from eq 11a is expressed in the form of the convolution
\[ N_{\text{long}}(\tilde{\Omega}) = \frac{T}{\mu} n_{\text{det}} \int d\omega S(\omega) \tilde{d}(\omega - \tilde{\Omega}) \] (12)
where \( \tilde{\omega} \) and \( \tilde{\Omega} \) are in units of \( \mu/\hbar \). Measuring the atom number at a single value of \( \Omega \), the current noise spectrum is readily obtained around this frequency with a kilohertz resolution, i.e., averaged in the bandwidth of the BEC chemical potential \( \mu/\hbar \). By fine-tuning \( \Omega \) via the field \( B_{\text{offs}} \) and using deconvolution, the spectrum \( S(\omega) \) can be deduced with a much higher resolution. We recall that \( \Omega \) is a frequency relative to the CNT vibrational frequency \( \omega_{\text{cnt}} \). Hence it can be set to both positive and negative values, which is substantial to the quantum galvanometer. Finally we note that the maximum frequency range of \( S(\omega) \) that can be accessed by the method is limited by the vibrational frequency \( \omega_{\text{cnt}} \) to be conform with the rotating wave approximation.

The integral norm of \( S(\omega) \) is \( \langle \tilde{F} \rangle \). Then, separating the \( \Omega \)-dependence given by the convolution of normalized spectral functions, the coupling strength is on the order of \( T \eta_{\text{det}}(\tilde{F}) / \mu \).

For a numerical estimate, consider a system described in Figure 3, where an \( L = 2 \mu \) CNT is oscillating with an amplitude of \( a = 10 \mu \), at a distance \( y_0 = 4 \mu \) from the BEC, in which case \( U^2 \approx 0.4 \). For a measurement time \( T \approx 1 \) s that conforms with the BEC lifetime in atomchip microtraps, a single atom detected, \( N_{\text{long}} = 1 \), corresponds to quantum fluctuations of the current on the order of \( \langle \tilde{F} \rangle^{1/2} \approx 1 \mu A \).
We note that the thermal magnetic near-field noise, which is present in the case of room-temperature atomchips due to the trapping wires or metallic coatings on the substrate, has high frequency components resonant with the hyperfine splitting and thus induces spin flips.\textsuperscript{33–36} This parasitic effect must be suppressed by moving the trap to sufficiently large distance from such field sources, while keeping the BEC–CNT distance $y_0$ in the micrometer range, or by using superconducting atomchips.\textsuperscript{37}

In conclusion, we have evaluated the coupling of trapped atoms to the magnetic field created by the electric current in a mechanically vibrating CNT. The modeled coupling was found to be strong enough to sense quantum features of the current noise spectrum by means of hyperfine-state-selective atom trapping wires or metallic coatings on the substrate, has high present in the case of room-temperature atomchips due to the parasitic effect must be suppressed by moving the trap to sufficiently large distance from such field sources, while keeping the BEC–CNT distance $y_0$ in the micrometer range, or by using superconducting atomchips.\textsuperscript{37}

This work was supported by the Hungarian National Office for Research and Technology under the contract ERC_HU_09 OPTOMECH, the Hungarian Academy of Sciences (Lendület Program, LP2011-016), and the Hungarian Scientific Research Fund (OTKA) under Contract No. K83858. J.F. acknowledges support by the BMBF (NanoFutur 03XS006) and the DFG SFB TRR21.

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